1-4 Properties of Numbers

Equivalent Expressions represent the same number. The following properties allow you to write an equivalent expression for a given expression.

Additive Identity 0	For any number a, $a + 0 = a$. (value remains the same) $3+0=3$
Additive Inverse the opposite	For any number a, $a + (-a) = 0$. (values create a zero pair) $3+(-3)=0$ $-5+5=0$
Multiplicative Identity 1	For any number a, $a \bullet 1 = a$. (value remains the same) $3\cdot 1=3$
Multiplicative Property of 0	For any number a, $a \bullet 0 = 0$. (product always = 0) $3\cdot 0=0$
Multiplicative Inverse Property reciprocal **(2 numbers whose product =1)**	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \bullet \frac{b}{a} = 1$. $\frac{3}{4}\cdot\frac{4}{3}=\frac{12}{12}=1$ $\frac{1}{2}\cdot\frac{2}{1}=\frac{2}{2}=1$ $\frac{-6}{1}\cdot\frac{1}{-6}=\frac{-6}{-6}=1$
Reflexive Property	For any number a, $a = a$. a number equals itself
Symmetric Property	For any numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$. $3+4=1+6$ if $3+4=5+2$ and $5+2=1+6$, then
Substitution Property Replacing	If $a = b$, then a may be replaced by b in any expression.

Evaluate $7(4-3) - 1 + 5 \cdot \frac{1}{5}$ Name the property used in each step. PEMDAS

Given

$7(4-3) - 1 + 5 \cdot \frac{1}{5} = 7(1) - 1 + 5 \cdot \frac{1}{5}$ → Substitution: $4 - 3 = 1$

$= 7 - 1 + 5 \cdot \frac{1}{5}$ → Multiplicative Identity: $7 \cdot 1 = 7$

$= 7 - 1 + 1$ → Multiplicative Inverse $5 \cdot \frac{1}{5} = 1$

$= 6 + 1$ → Substitution $7 - 1 = 6$

$= 7$ → Substitution $6 + 1 = 7$

As you evaluate an expression, you are constructing an argument using stated assumptions, definitions, and previously established results. The properties of numbers are valid reasons for steps in the argument.

→ Algebraic Proof

Your Turn:

Name the property used in each step.

1A. $2 \cdot 3 + (4 \cdot 2 - 8)$ given

$= 2 \cdot 3 + (8 - 8)$ Substitution $4 \cdot 2 = 8$

$8 + (-8)$

$= 2 \cdot 3 + (0)$ additive inverse $8 + (-8) = 0$

$= 6 + 0$ Substitution $2 \cdot 3 = 6$

$= 6$ additive identity $6 + 0 = 6$

1B. $7 \cdot \frac{1}{7} + 6(15 \div 3 - 5)$ given

$= 7 \cdot \frac{1}{7} + 6(5 - 5)$ Substitution $15 \div 3 = 5$

$5 + (-5)$

$= 7 \cdot \frac{1}{7} + 6(0)$ Additive Inverse $5 + (-5) = 0$

$= 1 + 6(0)$ multiplicative inverse $7 \cdot \frac{1}{7} = 1$

$= 1 + 0$ multiplicative Property of Zero $6(0) = 0$

$= 1$ Additive Identity $1 + 0 = 1$

The Commutative Properties state that the order in which you add or multiply numbers does not change their sum or product.

The Associative Properties state that the way you group three or more numbers when adding or multiplying does not change their sum or product.

Commutative Properties	For any numbers *a* and *b*, $a + b = b + a$ and $a \bullet b = b \bullet a$. 3 4 3+4 4+3 3·4 4·3 7 = 7 12 = 12
Associative Properties	For any numbers *a*, *b*, and *c*, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$. 3 4 5 (3+4)+5 3+(4+5) 7+5 3+9 12 = 12

(3·4)·5 3·(4·5)
12·5 3·20
60 = 60

Example 1:
Evaluate 6 • 2 • 3 • 5 using properties of numbers. Name the property used in each step.

6 • 2 • 3 • 5

= 6 • 3 • 2 • 5 Commutative Property(x)

= (6 • 3)(2 • 5) Associative Property (x)

= 18 • 10 Substitution.

= 180 Substitution.

The product is 180.

Example 2:
Evaluate 8.2 + 2.5 + 2.5 + 1.8 using properties of numbers. Name the property used in each step.

8.2 + 2.5 + 2.5 + 1.8

= 8.2 + 1.8 + 2.5 + 2.5 Commutative Prop. (+)

= (8.2 + 1.8) + (2.5 + 2.5) Associative Prop. (+)

= 10 + 5 Substitution.

= 15 Substitution.

The sum is 15.

Your turn: Evaluate 5 · 7 · 4 · 2 using the properties of numbers. Name the property used in each step.

5 · 7 · 4 · 2 = 5·2·7·4 Commutative (x)

= (5·2)·(7·4) associative (x)

= 10 · 28 Substitution 5·2=10, 7·4=28

= 280 substitution 10·28=280